Correcting microstructure comovement biases for integrated covariance

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**Abstract**

Finding a precise variance–covariance matrix is the building block of empirical finance. While microstructure-noise-robust methods for realized volatility are in the mainstream of financial econometrics, little if any attention has been devoted to estimating a noise-free realized covariance for overlooking the well-documented manifestation of commonality in market microstructure factors such as order flows, liquidity or herding. By documenting and recognizing this fact, we propose a microstructure-noise-free non-parametric covariance estimator to uncover the virtual integrated covariance. The estimator is easy to implement and performs admirably.

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1. Introduction

Finding a relevant variance–covariance matrix for assets is the building block of strategic allocation and modern risk management. For asset pricing, second moments necessarily play a key role within a world where investors are risk-averse, and covariance terms can outnumber variance terms when the universe consists of multiple risky assets. In an applied sense, an accurate assessment of the covariance and correlation of key financial variables will have important benefits for portfolio managers, risk managers, and financial regulators alike. Constructing realized volatility (hereafter referred to as RV) via high-frequency information has become popular since Andersen et al. (2001), Barndorff-Nielsen and Shephard (2002), and the recent multivariate generalization in Barndorff-Nielsen and Shephard (2004).
Understandably, the presence of market microstructure effects has pointed the microscope of econometric modelling at RV in order to focus on removing market microstructure noise contaminations (see Andersen et al. (2009) for a recent review). Moreover, when more than one asset is involved, the effect of non-synchronous trading among financial assets induces a severe bias towards zero in the covariance constructed from high-frequency data, and some recent treatments of the effect on realized covariances can be found in Hayashi and Yoshida (2005), Bandi and Russell (2007), Voev and Lunde (2007), Yeh et al. (2007), Barndorff-Nielsen et al. (2008), Zhang (in press) and Griffin and Oomen (in press) among others. Nonetheless, to our knowledge, the contemporaneous comovement in market microstructure noises has been overlooked in constructing realized covariance in the literature.

Our study attempts to highlight a new phenomenon that is distinct from the aforementioned effects from the empirical and methodological development points of view. Despite the well-documented evidence of commonality in order flows and liquidity attributed to program trading or style investing that might trigger correlated inventory fluctuations (see Chordia et al., 2000; Hasbrouck and Seppi, 2001), little if any attention has been devoted to knowing how these comovements in market microstructure noises influence the computation of realized covariance. To motivate this main idea and circumvent the effect of nonsynchronous trading, we compute the microstructure noise variations of four actively-traded equities on the NYSE, namely IBM, McDonalds, 3M, and Wal-Mart, from January 3, 2007 to April 30, 2009 using the estimator proposed by Hansen and Lunde (2006). The results are depicted in Fig. 1.

It is clear that each series of microstructure noise variations exhibits strong comovements across time and the correlations among these series of pairs in IBM/MCD, IBM/MMM, IBM/WMT, MCD/MMM, MCD/WMT, and MMM/WMT are relatively high, being 0.772, 0.801, 0.591, 0.818, 0.821, and 0.899, respectively. Given this observation, of equal interest would be a version of the noise-free estimator that takes account of the commonalities in these microstructure factors. Although mainstream bias-correction methods for RV are effective in being free from asset-specific market microstructure noise, they are silent in regard to this commonality. That is, merely performing bias-correction in the diagonal elements does not suffice to provide an accurate realized variance–covariance matrix for further financial applications since those off-diagonal covariance terms and the subsequent correlations may be seriously biased due to the noise comovements.

This article sheds light on two perspectives. We show how biased and misleading the classical realized volatility and covariances can be when market microstructure noises share commonality. We then propose a bias-corrected estimator based on the idea of Yeh and Wang (2008) to uncover the virtual integrated covariance (IC henceforth) among efficient returns. Through both the Monte Carlo experiments and an empirical study, we show that the easy-to-implement noise-free estimator performs admirably in delivering a more precise realized covariance matrix and realized correlation matrix as well in the absence of asynchronous trading.

2. Realized variance–covariance

Suppose the logarithm of a $k$-vector diffusion process $\mathbf{p}(t)$ follows:

$$d\mathbf{p}(t + \tau) = \mathbf{\mu}(t + \tau) + \mathbf{\Theta}(t + \tau)d\mathbf{W}(t + \tau), \quad 0 \leq \tau \leq 1, \quad t = 1, 2, \ldots,$$

(1)

where $\mathbf{\mu}(t + \tau)$ is the multivariate drift component, $\mathbf{\Theta}(t + \tau)$ is the instantaneous co-volatility matrix and $\mathbf{W}(t + \tau)$ is the standard multivariate Brownian motion. Assume that $\mathbf{\Theta}(t + \tau)$ is orthogonal to $\mathbf{W}(t + \tau)$. The instantaneous covariance matrix is $\mathbf{\Sigma}(t + \tau) = \mathbf{\Theta}(t + \tau)\mathbf{\Theta}(t + \tau)'$ with generic elements given by $\Sigma_{ij}(t + \tau)$.

Having becoming popular since the seminal pieces of Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002), the recent studies by Andersen et al. (2009) and Barndorff-Nielsen and Shephard

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1 We rule out the leverage effect, that is, correlation between the return innovations and volatility. The recent literature is seeking to loosen this assumption, and one should explore practical performance when this assumption is violated. However, this effect does not rule out the usefulness of our estimator. We thank the anonymous referee for pointing this out.
suggest estimating the quadratic covariation matrix for multivariate returns at time $t$, i.e.,
\[ \mathbf{R}_{t} \mathbf{R}(s) ds, \]
by taking the outer-product of the intraday return over the period, namely,
\[ \mathbf{RCOV}_{t} = \sum_{m} \mathbf{r}_{ij} \mathbf{r}_{ij}' \]
where $m \equiv 1/\delta$ is the sampling frequency and $\delta$ is the sampling interval. Barndorff-Nielsen and Shephard (2004) have shown that
\[ m^{1/2} \left[ \text{vech}(\mathbf{RCOV}_{t}) - \text{vech}\left( \int_{t-1}^{t} \Sigma(t + \tau) d\tau \right) \right] \rightarrow \mathcal{N}(\mathbf{0}, \mathbf{H}_{t}), \quad \text{as } m \rightarrow \infty, \]
where $\mathbf{H}_{t}$ is positive definite.
The subsequent inferential analysis of realized betas or realized correlation is built upon the aforementioned results. However, in reality, sampling high-frequency data induces market microstructure noise that causes these estimators to be biased. While vast numbers of studies propose different methods to accommodate or correct for the microstructure effects on these volatility estimators (see, for instance, the earlier works in Zhou (1996), Bandi and Russell (2006), Andersen et al. (2005), Zhang et al. (2005) and the references therein), these methods correct only for equity-specific microstructure noise without looking into the comovement of market microstructure noises among different equities.

Recent academic work on realized covariance has focused mainly on the issue of asynchronous trading (Hayashi and Yoshida, 2005; Bandi and Russell, 2007; Voev and Lunde, 2007; Yeh et al., 2007; Barndorff-Nielsen et al., 2008; Zhang, in press; Griffin and Oomen, in press). Nonetheless, relative to the extensive literature on the bias-correction for univariate RV and adjustment for the effect of microstructure noise on these estimators (see, for instance, the earlier works in Zhou (1996), Bandi and Russell (2006), Andersen et al. (2005), Zhang et al. (2005) and the references therein), these methods correct only for equity-specific microstructure noise without looking into the comovement of market microstructure noises among different equities.

3. Model setups

For asset $i$, we divide the $t$th trading day into $m$ sub-periods and express the observed price for each sub-period $j$ as

$$
\hat{p}_{i(t-1)+j\delta} = p_{i(t-1)+j\delta} + \vartheta_{i(t-1)+j\delta}, \quad j = 1, 2, \ldots, m,
$$

where $p_{i(t-1)+j\delta}$ is the efficient price, and $\vartheta_{i(t-1)+j\delta}$ is the market microstructure noise. As the efficient price process is measured with market microstructure noise, the observed return in the $j$th sub-period for day $t$ is

$$\hat{r}_{i(j,t)} = \ln(\hat{p}_{i(t-1)+j\delta}) - \ln(p_{i(t-1)+(j-1)\delta}),$$

$$= \ln(p_{i(t-1)+j\delta}) - \ln(p_{i(t-1)+(j-1)\delta}) + \eta_{i(j,t)} - \eta_{i(t-1)},$$

$$= r_{i(j,t)} + \epsilon_{i(j,t)},$$

where $\eta_{i(t)} = \ln(\vartheta_{i(t-1)+j\delta})$. Obviously, the microstructure noise induces a serially dependent measurement error in observed returns. Without loss of generality, we suppress time $t$ from the subscript $(j,t)$ as $j$. To motivate our approach, we use the typical assumptions that have been employed in the literature to maintain simplicity and gain insights into the newly-proposed estimator.

3.1. Assumptions for market microstructure noise

(a) The market microstructure noise $\eta_{ij}$ is i.i.d. mean zero.

(b) The microstructure noise $\eta_{ij}$ is independent of any price process.

(c) Noise variance is a finite constant, $\sigma_i^2 \equiv E[\eta_{i1}^2] < \infty$.

(d) Noise covariance is also a finite constant, $\phi_{12} \equiv E[\eta_{1j} \cdot \eta_{2j}] < \infty$.

Assumptions (a) and (c) regulate the noise to be non-autocorrelated and have a constant variance. It is assumed that orthogonality between noise and returns in (b) is common. Assumption (d) characterizes the empirical commonality among market microstructure noises.

4. Noise-free estimators for diagonal and off-diagonal terms

The diagonal and off-diagonal elements in $\Sigma(t + \tau)$ can be estimated by RV and CP as

$$\text{RV}_i(m) \equiv \sum_{j=1}^{m} r_{ij}^2 = \sum_{j=1}^{m} \hat{r}_{i1j}^2 + \sum_{j=1}^{m} r_{ij}\epsilon_{ij} + \sum_{j=1}^{m} \epsilon_{ij}^2, \quad i = 1, 2,$$

$$\text{CP}_{12}(m) \equiv \sum_{j=1}^{m} r_{1j}\hat{r}_{2j} = \sum_{j=1}^{m} r_{1j}\hat{r}_{2j} + \sum_{j=1}^{m} r_{1j}\epsilon_{2j} + \sum_{j=1}^{m} r_{2j}\epsilon_{1j} + \sum_{j=1}^{m} \epsilon_{1j}\epsilon_{2j}.$$
In the absence of market microstructure noise, i.e., $\eta_{ij} = 0$, the RV converges to IV and CP converges to IC as $m \to \infty$.

Nonetheless, we can verify under the assumptions for market microstructure noises that
\begin{align}
E_m[RV_i(m)] &= IV_i + 2m\omega_i^2, \\
E_m[CP_{1,2}(m)] &= IC_{1,2} + 2m\phi_{1,2}.
\end{align}

Both RV and CP are biased estimators for IV and IC. Yeh and Wang (2008) propose an easy bias-corrected estimator for IV through a simple linear combination of two RVs at distinct sampling frequencies, i.e.,
\[ RV_i^*(m_1, m_2) = \frac{\kappa RV_i(m_2) - RV_i(m_1)}{\kappa - 1}, \]
where $m_1 = km_2$.

This approach may at first seem to be close to the two-scale estimator by Zhang et al. (2005), but the spirit of the exact bias-correction as well as the finite sample performance of their estimator are quite different and, as it turns out, much simpler. The fact that they are using different sampling frequencies does not immediately result in an approach which is necessarily similar to the two-scale approach. In fact, the entire literature relies on identification at different frequencies by estimating the properties of the noise and the properties of the price process for the purpose of asymptotic bias-correcting (by subtracting the estimates of the noise variance) or asymptotic/finite sample MSE evaluation. It is a well-known fact that a large component of the finite sample MSE of some integrated variance estimators (including realized variance and, incidentally, the traditional two-scale estimator) is bias-induced. Instead, Yeh and Wang (2008) define realized variance measures with bias components which nicely cancel each other out when appropriately re-weighted. Contrary to the traditional two-scale estimator of Zhang et al. (2005) that is asymptotically unbiased, their exact bias-corrected realized variance is unbiased in finite samples.

In parallel with their work that shows the nice properties of RV*, we extend the same idea for IC through a linear combination of CP$_{1,2}$($m_1$) and CP$_{1,2}$($m_2$) as
\begin{align}
CP_{1,2}^\ast(m_1, m_2) &= \frac{\kappa CP_{1,2}(m_2) - CP_{1,2}(m_1)}{\kappa - 1},
\end{align}
where $m_1 = km_2$. Based on the assumptions and the pre-specified $m_1$ and $m_2$, we again arrive at an unbiased property as follows
\begin{align}
E_{m_1, m_2}[CP_{1,2}^\ast(m_1, m_2)] &= \frac{1}{\kappa - 1} \left(\kappa(IC_{1,2} + 2m_2\phi_{1,2}) - (IC_{1,2} + 2m_1\phi_{1,2})\right) = IC_{1,2}.
\end{align}

The desired comovement-bias-free estimator for IC is readily available and we perform Monte Carlo experiments and discuss the empirical relevance in the next section.

5. Monte Carlo experiments and empirical relevance

To demonstrate the superior performance of the bias-corrected estimators over the conventional ones using discrete sampling under various market microstructure scenarios, we conduct Monte Carlo experiments following the setups in Brandt and Diebold (2006) as an illustration. We consider 2000 trading days with 24 trading hours being divided into $m$ regularly spaced high-frequency returns each day. Supposing that $p_{i0} = 1, i = 1, 2$, the data generation process (DGP) at each instant is
\begin{align}
\ln(p_{it+j}) &= \ln(p_{it+j-1}) + \sigma \sqrt{250/mz_{it+j}}, \quad i = 1, 2,
\end{align}
where $j = 1, 2, \ldots, m$, $t = 1, 2, \ldots, 2000$, and $z_{it+j}$ are standard normal innovations with a covariance of 0.9. Moreover, we let $\sigma^2 = 0.0006$, which amounts to an annualized (250 trading days within a year) volatility of 15%, and a correlation efficient between returns of 0.4.

We construct both the typical and bias-corrected version of realized volatility, covariance, and correlation within different scenarios to highlight the extant discussions.
• Scenario I: Merton’s Utopia without market microstructure noise.
• Scenario II: Mutually orthogonal market microstructure noises.
• Scenario III: Commonality in market microstructure noises.

We simulate the bid-ask bounce as the market microstructure noise. With the bid-ask spread being roughly 0.0005, we generate the noises as

$$\eta_{i,t,j} = 0.00025 I_{i,t,j},$$

where $$I_{i,t,j} \sim \text{Bernoulli}[1/2]$$, $$I_{1,t,j}$$ and $$I_{2,t,j}$$ are orthogonal in Scenario II and correlated with 0.5 for Scenario III. Moreover, we regard 0.0001 as the tick size and round the observed prices down or up to the nearest tick. We examine five sampling frequencies at 1-, 5-, 10-, 20-, and 40-min intervals, respectively. We replicate the procedure 1000 times and report the means, standard deviations, and RMSE of the estimates obtained in Table 1.

5.1. Monte Carlo experiment results

In Panel A of Merton’s Utopia that characterizes the world of Barndorff-Nielsen and Shephard (2004), all estimates are in line with the simulation parameters and become more efficient as $$m \to \infty$$, whereas the bias-corrected estimates of RV* and CP* are slightly less efficient than the conventional ones in terms of RMSE.

The results for Scenarios II and III with orthogonal or comoving market microstructure noises are in Panels B and C, respectively. In Panel B, based on the previous arguments, it might have been anticipated that RV is severely upward-biased when sampled at higher frequencies. Since the bid-ask indicators between assets 1 and 2 are uncorrelated, the estimated covariance using CP is immune to the mutually orthogonal noises. Evidently, as volatility is over-estimated, the realized correlation is

Table 1

Typical and bias-corrected estimates for different scenarios. We perform the simulation using DGP in Eq. (15) with an annualized volatility of 15%, covariance of 0.9, and correlation of 0.4. We compute traditional realized estimates as shown in the left panels based on Eqs. (8) and (9). The right panels are the computed noise-free realized estimates based on Eqs. (12) and (13). The experiments obviously show the noise-attributed upward biases in RV and the overestimation of CP when there are contemporaneous comovements in noises.

<table>
<thead>
<tr>
<th>Sampling freq.</th>
<th>Volatility</th>
<th>Covariance</th>
<th>Sampling freq.</th>
<th>Volatility</th>
<th>Covariance</th>
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<tr>
<td>Mean</td>
<td>SD</td>
<td>RMSE</td>
<td>Mean</td>
<td>SD</td>
<td>RMSE</td>
</tr>
<tr>
<td>Panel A. Merton’s Utopia without noise</td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>1-min</td>
<td>15.006</td>
<td>0.279</td>
<td>0.279</td>
<td>0.900</td>
<td>0.063</td>
</tr>
<tr>
<td>5-min</td>
<td>15.008</td>
<td>0.628</td>
<td>0.628</td>
<td>0.898</td>
<td>0.146</td>
</tr>
<tr>
<td>10-min</td>
<td>14.948</td>
<td>0.884</td>
<td>0.884</td>
<td>0.898</td>
<td>0.197</td>
</tr>
<tr>
<td>20-min</td>
<td>14.947</td>
<td>1.244</td>
<td>1.245</td>
<td>0.899</td>
<td>0.272</td>
</tr>
<tr>
<td>40-min</td>
<td>14.844</td>
<td>1.763</td>
<td>1.770</td>
<td>0.889</td>
<td>0.414</td>
</tr>
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<td>Panel B. Cross-independent market microstructure noises</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-min</td>
<td>26.088</td>
<td>0.436</td>
<td>11.097</td>
<td>0.897</td>
<td>1.109</td>
</tr>
<tr>
<td>5-min</td>
<td>17.793</td>
<td>0.728</td>
<td>2.886</td>
<td>0.896</td>
<td>0.202</td>
</tr>
<tr>
<td>10-min</td>
<td>16.423</td>
<td>0.945</td>
<td>1.708</td>
<td>0.897</td>
<td>0.236</td>
</tr>
<tr>
<td>20-min</td>
<td>15.704</td>
<td>1.304</td>
<td>1.482</td>
<td>0.902</td>
<td>0.296</td>
</tr>
<tr>
<td>40-min</td>
<td>15.233</td>
<td>1.822</td>
<td>1.837</td>
<td>0.907</td>
<td>0.410</td>
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<td>Panel C. Microstructure noises with contemporaneous comovements</td>
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<td></td>
<td></td>
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<tr>
<td>1-min</td>
<td>26.100</td>
<td>0.452</td>
<td>11.109</td>
<td>3.153</td>
<td>0.201</td>
</tr>
<tr>
<td>5-min</td>
<td>17.781</td>
<td>0.728</td>
<td>2.874</td>
<td>1.355</td>
<td>0.210</td>
</tr>
<tr>
<td>10-min</td>
<td>16.397</td>
<td>0.946</td>
<td>1.687</td>
<td>1.127</td>
<td>0.242</td>
</tr>
<tr>
<td>20-min</td>
<td>15.664</td>
<td>1.299</td>
<td>1.459</td>
<td>1.015</td>
<td>0.317</td>
</tr>
<tr>
<td>40-min</td>
<td>15.203</td>
<td>1.810</td>
<td>1.821</td>
<td>0.944</td>
<td>0.435</td>
</tr>
</tbody>
</table>

undoubtedly underestimated in particular for higher frequencies. However, the noise-free RV* and CP* based on two frequencies, estimates for volatility and covariance perform reasonably well and increasing m will not deteriorate but will instead enhance their RMSEs due to more efficiency gains. Nonetheless, we demonstrate in what follows that overlooking the contemporaneous comovement among noises leads to serious bias when assessing the covariance.

Panel C summarizes the results where not only are market microstructure noises allowed, but the market microstructure noises are contemporaneously correlated with a correlation of 0.5. RV and CP are biased in the presence of noises with comovement. The upward biases in RV and CP increase sharply with m. Alternatively, replacing RV and CP with the bias-corrected RV* and CP* results in noise-free estimates. Interestingly and perhaps not surprisingly, Panel C again reveals an intriguing property of CP*: imposing a linear combination between two estimators sampled at different frequencies helps get rid of the microstructure effects, particularly when noises share contemporaneous comovements.

5.2. Empirical results

To disentangle the performance of the estimators in a more practically sensible noise structure and to set aside the potential issue of asynchronous trading, we examine our newly-proposed estimators of RV* and CP* as well as the typical estimators for four equities actively traded on the NYSE, i.e., IBM, McDonalds, 3M, and Wal-Mart, over the period from January 3, 2007 to April 30, 2009.

Based on the noise variation and commonality disclosed in Fig. 1, the presence of biases in both estimates of variance and covariance based upon the typical RV and CP might have been anticipated. Our empirical results in Panel A of Table 2 indeed reveal the case where each element of the variance–covariance matrix in the left panel is overestimated as compared with the corresponding element in the bias-corrected estimates using our estimators in the right panel. Perhaps not surprisingly, as shown in Panel B of Table 2, the realized correlation matrix that is obtained based on the typical estimators is consequently biased in the presence of microstructure noise comovements, if compared with the right panel of the bias-corrected realized correlations. The observed downward bias in the realized correlation in the left Panel B of Table 2 can be explained by the fact that the inflation due to noise in the realized variance is more severe than the inflation contributed by the commonality in the realized covariance.

Table 2
Realized covariance and correlation matrix – empirical relevance of the bias-correction. We consider four actively-traded stocks, i.e., IBM, MCD, MMM, and WMT, over the period from January 3, 2007 to April 30, 2009 to circumvent the effect of non-synchronous trading. We construct the realized covariance matrix in Panel A based on the noise-polluted RV(78) and CP(78); and the bias-corrected RVi(195, 39) and CPi(195, 39). The corresponding realized correlation matrix for both sets of estimators are presented in Panel B. All values of the covariances are multiplied by 104. The traditional noise-polluted estimates obtained at 5-min sampling frequency are generally upwardly-biased, as compared to the re-weighted bias-corrected ones based on 2-min and 10-min estimates. As the inflation due to noise in the realized variance is more serious than that in the covariance, the realized correlation obtained tends to be biased downward if no bias-adjustment is applied.

<table>
<thead>
<tr>
<th></th>
<th>Traditional estimates by 5-min</th>
<th>Bias-corrected estimates by 2-min and 10-min</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>IBM</td>
<td>MCD</td>
</tr>
<tr>
<td>Panel A. Realized covariance matrix estimates</td>
<td></td>
<td></td>
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<tr>
<td>IBM</td>
<td>3.854</td>
<td>–</td>
</tr>
<tr>
<td>MCD</td>
<td>1.587</td>
<td>3.445</td>
</tr>
<tr>
<td>MMM</td>
<td>1.898</td>
<td>1.600</td>
</tr>
<tr>
<td>WMT</td>
<td>1.549</td>
<td>1.444</td>
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<tr>
<td>Panel B. Realized correlation matrix estimates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IBM</td>
<td>1.000</td>
<td>–</td>
</tr>
<tr>
<td>MCD</td>
<td>0.435</td>
<td>1.000</td>
</tr>
<tr>
<td>MMM</td>
<td>0.498</td>
<td>0.444</td>
</tr>
<tr>
<td>WMT</td>
<td>0.435</td>
<td>0.429</td>
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</tbody>
</table>
6. Conclusions

This paper provides a motivation for a new empirical regularity, commonality among microstructure effects, that is distinct from the usual microstructure noise of a specific asset or the effect of non-synchronous trading, to be accommodated in the estimation of realized covariance. This paper proposes a bias-free estimator for IC and compares the performance of this bias-corrected estimator with the traditional estimator used in Monte Carlo experiments and an empirical study. By using high frequency transaction data for four equities actively traded on the NYSE to control for the effect of asynchronous trading, our results suggest that overlooking the existence of commonality in market microstructure effects when computing a realized covariance matrix can be dangerous. Taken as a whole, our newly-proposed easy-to-implement estimator not only offers a nice remedy to this issue through its finite-sample justification but also performs well empirically.

Our assumptions might be restrictive in certain circumstances, but they are certainly bound to capture important first-order effects in the data. Although beyond the scope of the present paper, a further examination of our estimator under a weaker set of assumptions or a more general set of noise structures can be conducted. However, the generality of this result is unknown and currently under investigation.

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